CHAPTER

31

Worm Gears

31.1 Introduction

The worm gears are widely used for transmitting power at high velocity ratios between non-intersecting shafts that are generally, but not necessarily, at right angles. It can give velocity ratios as high as 300 : 1 or more in a single step in a minimum of space, but it has a lower efficiency. The worm gearing is mostly used as a speed reducer, which consists of worm and a worm wheel or gear. The worm (which is the driving member) is usually of a cylindrical form having threads of the same shape as that of an involute rack. The threads of the worm may be left handed or right handed and single or multiple threads. The worm wheel or gear (which is the driven member) is similar to a helical gear with a face curved to conform to the shape of the worm. The worm is generally made of steel while the worm gear is made of bronze or cast iron for light service.

The worm gearing is classified as non-interchangeable, because a worm wheel cut with a hob of one diameter will not operate satisfactorily with a worm of different diameter, even if the thread pitch is same.

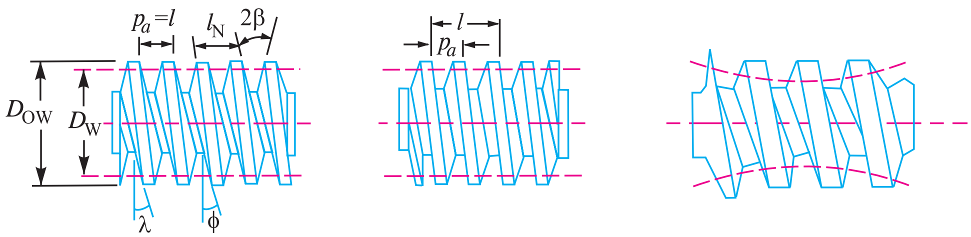
31.2 Types of Worms

The following are the two types of worms :

1. Cylindrical or straight worm, and
2. Cone or double enveloping worm.

The ***cylindrical*** or ***straight worm***, as shown in Fig. 31.1 (*a*), is most commonly used. The shape of the thread is involute helicoid of pressure angle 14 ½° for single and double threaded worms and 20° for triple and quadruple threaded worms. The worm threads are cut by a straight sided milling cutter having its diameter not less than the outside diameter of worm or greater than 1.25 times the outside diameter of worm.

The ***cone*** or ***double enveloping worm***, as shown in Fig. 31.1 (*b*), is used to some extent, but it requires extremely accurate alignment.



Single threaded. Double threaded.

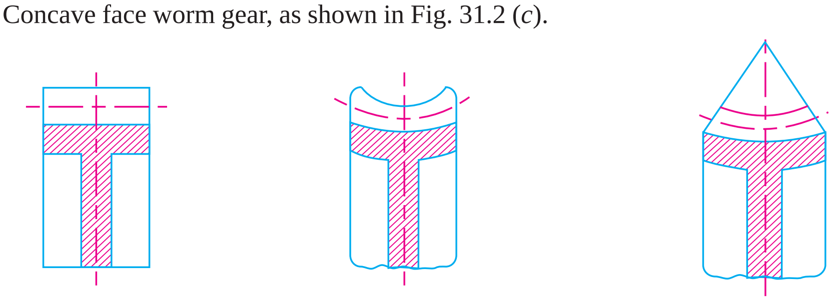
(*a*) Cylindrical or straight worm. (*b*) Cone or double enveloping worm.

Fig. 31.1. Types of worms.

31.3 Types of Worm Gears

The following three types of worm gears are important from the subject point of view :

1. Straight face worm gear, as shown in Fig. 31.2 (*a*),
2. Hobbed straight face worm gear, as shown in Fig. 31.2 (*b*), and
3. Concave face worm gear, as shown in Fig. 31.2 (*c*).



(*a*) Straight face. (*b*) Hobbed straight face. (*c*) Concave face.

Fig. 31.2. Types of worms gears.

The ***straight face worm gear*** is like a helical gear in which the straight teeth are cut with a form cutter. Since it has only point contact with the worm thread, therefore it is used for light service.

The ***hobbed straight face worm gear*** is also used for light service but its teeth are cut with a hob, after which the outer surface is turned.

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The ***concave face worm gear*** is the accepted standard form and is used for all heavy service and general industrial uses. The teeth of this gear are cut with a hob of the same pitch diameter as the mating worm to increase the contact area.

Worm gear is used mostly where the power source operates at a high speed and output is at a slow speed with high torque. It is also used in some cars and trucks.

31.4 Terms used in Worm Gearing

The worm and worm gear in mesh is shown in Fig. 31.3.

The following terms, in connection with the worm gearing, are important from the subject point of view :

1. *Axial pitch.* It is also known as ***linear pitch*** of a worm. It is the distance measured axially (*i.e.* parallel to the axis of worm) from a point on one thread to the corresponding point on the adjacent thread on the worm, as shown in Fig. 31.3. It may be noted that the axial pitch (p_a) of a worm is equal to the circular pitch (p_c) of the mating worm gear, when the shafts are at right angles.

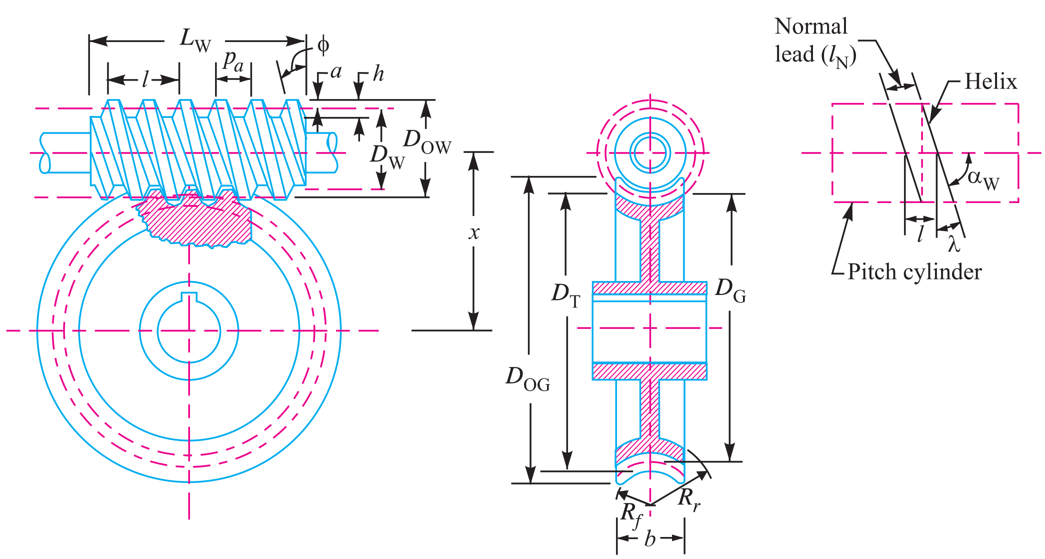

Fig. 31.3 . Worm and Worm gear.

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2. *Lead.* It is the linear distance through which a point on a thread moves ahead in one revolution of the worm. For single start threads, lead is equal to the axial pitch, but for multiple start threads, lead is equal to the product of axial pitch and number of starts. Mathematically,

Lead, $l = p_a \, . \, n$

where p_a = Axial pitch ; and n = Number of starts.

3. *Lead angle.* It is the angle between the tangent to the thread helix on the pitch cylinder and the plane normal to the axis of the worm. It is denoted by λ.

A little consideration will show that if one complete turn of a worm thread be imagined to be unwound from the body of the worm, it will form an inclined plane whose base is equal to the pitch circumference of the worm and altitude equal to lead of the worm, as shown in Fig. 31.4.

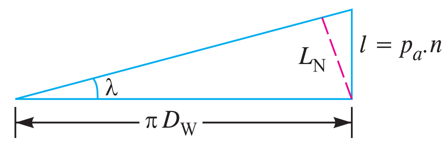


Fig. 31.4. Development of a helix thread.

From the geometry of the figure, we find that

$$\tan \lambda = \frac{\text{Lead of the worm}}{\text{Pitch circumference of the worm}}$$

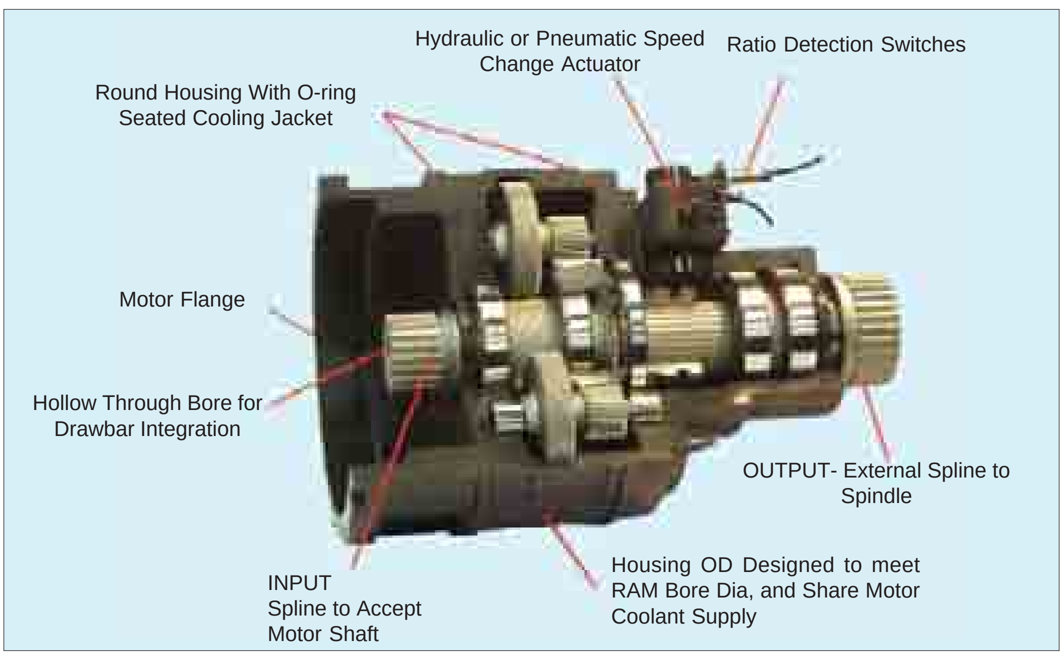
$$= \frac{l}{\pi D_W} = \frac{p_a \, . n}{\pi D_W} \qquad \ldots (\because \; l = p_a \, . \, n)$$

$$= \frac{p_c \, . n}{\pi D_W} = \frac{\pi \, m . n}{\pi D_W} = \frac{m . n}{D_W} \qquad \ldots (\because \; p_a = p_c \text{ ; and } p_c = \pi \, m)$$

where m = Module, and

D_W = Pitch circle diameter of worm.

The lead angle (λ) may vary from 9° to 45°. It has been shown by F.A. Halsey that a lead angle less than 9° results in rapid wear and the safe value of λ is 12½°.



Model of sun and planet gears.

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For a compact design, the lead angle may be determined by the following relation, *i.e.*

$$\tan \lambda = \left(\frac{N_G}{N_W}\right)^{1/3},$$

where N_G is the speed of the worm gear and N_W is the speed of the worm.

4. *Tooth pressure angle.* It is measured in a plane containing the axis of the worm and is equal to one-half the thread profile angle as shown in Fig. 31.3.

The following table shows the recommended values of lead angle (λ) and tooth pressure angle (φ).

Table 31.1. Recommended values of lead angle and pressure angle.

Lead angle (λ) in degrees	0 – 16	16 – 25	25 – 35	35 – 45
Pressure angle(φ) in degrees	14½	20	25	30

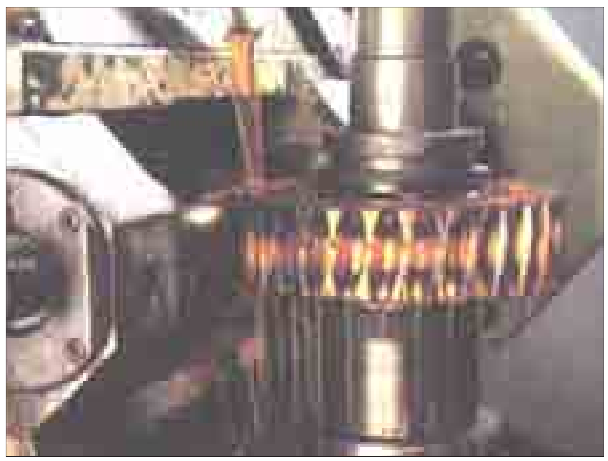
For automotive applications, the pressure angle of 30° is recommended to obtain a high efficiency and to permit overhauling.

5. *Normal pitch.* It is the distance measured along the normal to the threads between two corresponding points on two adjacent threads of the worm. Mathematically,

Normal pitch, $p_N = p_a . \cos \lambda$

Note. The term normal pitch is used for a worm having single start threads. In case of a worm having multiple start threads, the term normal lead (l_N) is used, such that

$$l_N = l . \cos \lambda$$



Worm gear teeth generation on gear hobbing machine.

6. *Helix angle.* It is the angle between the tangent to the thread helix on the pitch cylinder and the axis of the worm. It is denoted by α_W, in Fig. 31.3. The worm helix angle is the complement of worm lead angle, *i.e.*

$$\alpha_W + \lambda = 90°$$

It may be noted that the helix angle on the worm is generally quite large and that on the worm gear is very small. Thus, it is usual to specify the lead angle (λ) on the worm and helix angle (α_G) on the worm gear. These two angles are equal for a 90° shaft angle.

7. *Velocity ratio.* It is the ratio of the speed of worm (N_W) in r.p.m. to the speed of the worm gear (N_G) in r.p.m. Mathematically, velocity ratio,

$$V.R. = \frac{N_W}{N_G}$$

Let l = Lead of the worm, and

D_G = Pitch circle diameter of the worm gear.

We know that linear velocity of the worm,

$$v_W = \frac{l . N_W}{60}$$

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and linear velocity of the worm gear,

$$v_G = \frac{\pi\, D_G\; N_G}{60}$$

Since the linear velocity of the worm and worm gear are equal, therefore

$$\frac{l\,.N_W}{60} = \frac{\pi\, D_G\,.N_G}{60} \quad \text{or} \quad \frac{N_W}{N_G} = \frac{\pi\, D_G}{l}$$

We know that pitch circle diameter of the worm gear,

$$D_G = m\,.\,T_G$$

where m is the module and T_G is the number of teeth on the worm gear.

$$\therefore \qquad V.R. = \frac{N_W}{N_G} = \frac{\pi\, D_G}{l} = \frac{\pi\, m.T_G}{l}$$

$$= \frac{p_c\,.T_G}{l} = \frac{p_a\,.T_G}{p_a\,.n} = \frac{T_G}{n} \qquad \ldots (\because p_c = \pi\, m = p_a\,;\text{ and } l = p_a\,.\,n)$$

where n = Number of starts of the worm.

From above, we see that velocity ratio may also be defined as the ratio of number of teeth on the worm gear to the number of starts of the worm.

The following table shows the number of starts to be used on the worm for the different velocity ratios :

Table 31.2. Number of starts to be used on the worm for different velocity ratios.

Velocity ratio (*V.R.*)	36 and above	12 to 36	8 to 12	6 to 12	4 to 10
Number of starts or threads on the worm ($n = T_w$)	Single	Double	Triple	Quadruple	Sextuple

31.5 Proportions for Worms

The following table shows the various porportions for worms in terms of the axial or circular pitch (p_c) in mm.

Table 31.3. Proportions for worm.

S. No.	*Particulars*	*Single and double threaded worms*	*Triple and quadruple threaded worms*
1.	Normal pressure angle (φ)	14½°	20°
2.	Pitch circle diameter for worms integral with the shaft	2.35 p_c + 10 mm	2.35 p_c + 10 mm
3.	Pitch circle diameter for worms bored to fit over the shaft	2.4 p_c + 28 mm	2.4 p_c + 28 mm
4.	Maximum bore for shaft	p_c + 13.5 mm	p_c + 13.5 mm
5.	Hub diameter	1.66 p_c + 25 mm	1.726 p_c + 25 mm
6.	Face length (L_W)	p_c (4.5 + 0.02 T_W)	p_c (4.5 + 0.02 T_W)
7.	Depth of tooth (h)	0.686 p_c	0.623 p_c
8.	Addendum (a)	0.318 p_c	0.286 p_c

Notes: 1. The pitch circle diameter of the worm (D_W) in terms of the centre distance between the shafts (x) may be taken as follows :

$$D_W = \frac{(x)^{0.875}}{1.416} \qquad \ldots \text{(when } x \text{ is in mm)}$$

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2. The pitch circle diameter of the worm (D_W) may also be taken as

$$D_W = 3\,p_c, \text{ where } p_c \text{ is the axial or circular pitch.}$$

3. The face length (or length of the threaded portion) of the worm should be increased by 25 to 30 mm for the feed marks produced by the vibrating grinding wheel as it leaves the thread root.

31.6 Proportions for Worm Gear

The following table shows the various proportions for worm gears in terms of circular pitch (p_c) in mm.

Table 31.4. Proportions for worm gear.

S. No.	*Particulars*	*Single and double threads*	*Triple and quadruple threads*
1.	Normal pressure angle (ϕ)	14½°	20°
2.	Outside diameter (D_{OG})	$D_G + 1.0135\,p_c$	$D_G + 0.8903\,p_c$
3.	Throat diameter (D_T)	$D_G + 0.636\,p_c$	$D_G + 0.572\,p_c$
4.	Face width (b)	$2.38\,p_c + 6.5$ mm	$2.15\,p_c + 5$ mm
5.	Radius of gear face (R_f)	$0.882\,p_c + 14$ mm	$0.914\,p_c + 14$ mm
6.	Radius of gear rim (R_r)	$2.2\,p_c + 14$ mm	$2.1\,p_c + 14$ mm

31.7 Efficiency of Worm Gearing

The efficiency of worm gearing may be defined as the ratio of work done by the worm gear to the work done by the worm.

Mathematically, the efficiency of worm gearing is given by

$$\eta = \frac{\tan\lambda\,(\cos\phi - \mu\tan\lambda)}{\cos\phi\tan\lambda + \mu} \qquad ...(i)$$

where ϕ = Normal pressure angle,

μ = Coefficient of friction, and

λ = Lead angle.

The efficiency is maximum, when

$$\tan\lambda = \sqrt{1+\mu^2} - \mu$$

In order to find the approximate value of the efficiency, assuming square threads, the following relation may be used :

$$\text{Efficiency, } \eta = \frac{\tan\lambda\,(1-\mu\tan\lambda)}{\tan\lambda+\mu}$$

$$= \frac{1-\mu\tan\lambda}{1+\mu/\tan\lambda}$$

$$= \frac{\tan\lambda}{\tan(\lambda+\phi_1)}$$

...(Substituting in equation (*i*), $\phi = 0$, for square threads)

where ϕ_1 = Angle of friction, such that $\tan\phi_1 = \mu$.

A gear-cutting machine is used to cut gears.

The coefficient of friction varies with the speed, reaching a minimum value of 0.015 at a rubbing speed $\left(v_r = \frac{\pi\, D_W \,.\, N_W}{\cos \lambda}\right)$ between 100 and 165 m/min. For a speed below 10 m/min, take $\mu = 0.015$. The following empirical relations may be used to find the value of μ, *i.e.*

$$\mu = \frac{0.275}{(v_r)^{0.25}}, \text{ for rubbing speeds between 12 and 180 m/min}$$

$$= 0.025 + \frac{v_r}{18000} \text{ for rubbing speed more than 180 m/min}$$

Note : If the efficiency of worm gearing is less than 50%, then the worm gearing is said to be ***self locking,*** *i.e.* it cannot be driven by applying a torque to the wheel. This property of self locking is desirable in some applications such as hoisting machinery.

Hardened and ground worm shaft and worm wheel pair

Example 31.1. *A triple threaded worm has teeth of 6 mm module and pitch circle diameter of 50 mm. If the worm gear has 30 teeth of 14½° and the coefficient of friction of the worm gearing is 0.05, find 1. the lead angle of the worm, 2. velocity ratio, 3. centre distance, and 4. efficiency of the worm gearing.*

Solution. Given : $n = 3$; $m = 6$; $D_W = 50$ mm ; $T_G = 30$; $\phi = 14.5°$; $\mu = 0.05$.

1. Lead angle of the worm

Let λ = Lead angle of the worm.

We know that $\tan \lambda = \frac{m.n}{D_W} = \frac{6 \times 3}{50} = 0.36$

$\therefore$ $\lambda = \tan^{-1} (0.36) = 19.8°$ **Ans.**

2. Velocity ratio

We know that velocity ratio,

$$V.R. = T_G / n = 30 / 3 = 10 \text{ **Ans.**}$$

3. Centre distance

We know that pitch circle diameter of the worm gear

$$D_G = m.T_G = 6 \times 30 = 180 \text{ mm}$$

∴ Centre distance,

$$x = \frac{D_W + D_G}{2} = \frac{50 + 180}{2} = 115 \text{ mm}$$ **Ans.**

4. Efficiency of the worm gearing

We know that efficiency of the worm gearing.

$$\eta = \frac{\tan \lambda\, (\cos \phi - \mu \tan \lambda)}{\cos \phi \,.\, \tan \lambda + \mu}$$

$$= \frac{\tan 19.8° \,(\cos 14.5° - 0.05 \times \tan 19.8°)}{\cos 14.5° \times \tan 19.8° + 0.05}$$

$$= \frac{0.36\,(0.9681 - 0.05 \times 0.36)}{0.9681 \times 0.36 + 0.05} = \frac{0.342}{0.3985} = 0.858 \text{ or } 85.8\%$$ **Ans.**

Note : The approximate value of the efficiency assuming square threads is

$$\eta = \frac{1 - \mu \tan \lambda}{1 + \mu / \tan \lambda} = \frac{1 - 0.05 \times 0.36}{1 + 0.05 / 0.36} = \frac{0.982}{1.139} = 0.86 \text{ or } 86\% \quad \textbf{Ans.}$$

31.8 Strength of Worm Gear Teeth

In finding the tooth size and strength, it is safe to assume that the teeth of worm gear are always weaker than the threads of the worm. In worm gearing, two or more teeth are usually in contact, but due to uncertainty of load distribution among themselves it is assumed that the load is transmitted by one tooth only. We know that according to Lewis equation,

$$W_T = (\sigma_o \, . \, C_v) \, b . \pi \, m \, . \, y$$

where W_T = Permissible tangential tooth load or beam strength of gear tooth,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y = Tooth form factor or Lewis factor.

Notes : 1. The velocity factor is given by

$$C_v = \frac{6}{6 + v}, \text{ where } v \text{ is the peripheral velocity of the worm gear in m/s.}$$

2. The tooth form factor or Lewis factor (y) may be obtained in the similar manner as discussed in spur gears (Art. 28.17), *i.e.*

$$y = 0.124 - \frac{0.684}{T_G}, \text{ for } 14\tfrac{1}{2}° \text{ involute teeth.}$$

$$= 0.154 - \frac{0.912}{T_G}, \text{ for } 20° \text{ involute teeth.}$$

3. The dynamic tooth load on the worm gear is given by

$$W_D = \frac{W_T}{C_v} = W_T \left(\frac{6 + v}{6} \right)$$

where W_T = Actual tangential load on the tooth.

The dynamic load need not to be calculated because it is not so severe due to the sliding action between the worm and worm gear.

4. The static tooth load or endurance strength of the tooth (W_S) may also be obtained in the similar manner as discussed in spur gears (Art. 28.20), *i.e.*

$$W_S = \sigma_e . b \, \pi \, m . y$$

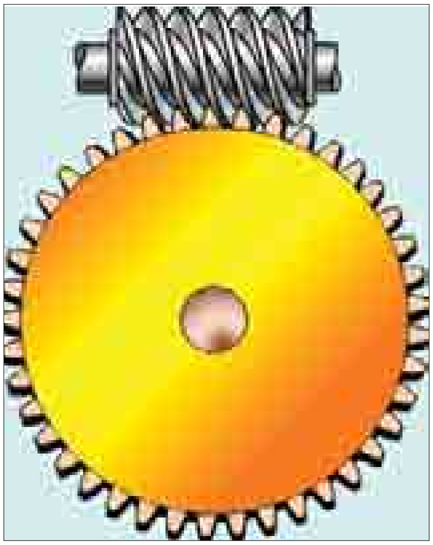
where σ_e = Flexural endurance limit. Its value may be taken as 84 MPa for cast iron and 168 MPa for phosphor bronze gears.

31.9 Wear Tooth Load for Worm Gear

The limiting or maximum load for wear (W_W) is given by

$$W_W = D_G \, . \, b \, . \, K$$

where D_G = Pitch circle diameter of the worm gear,



Worm gear assembly.

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b = Face width of the worm gear, and

K = Load stress factor (also known as material combination factor).

The load stress factor depends upon the combination of materials used for the worm and worm gear. The following table shows the values of load stress factor for different combination of worm and worm gear materials.

Table 31.5. Values of load stress factor (*K*).

S.No.	*Material*		*Load stress factor (K)* *N/mm²*
	Worm	*Worm gear*	
1.	Steel (B.H.N. 250)	Phosphor bronze	0.415
2.	Hardened steel	Cast iron	0.345
3.	Hardened steel	Phosphor bronze	0.550
4.	Hardened steel	Chilled phosphor bronze	0.830
5.	Hardened steel	Antimony bronze	0.830
6.	Cast iron	Phosphor bronze	1.035

Note : The value of K given in the above table are suitable for lead angles upto 10°. For lead angles between 10° and 25°, the values of K should be increased by 25 per cent and for lead angles greater than 25°, increase the value of K by 50 per cent.

31.10 Thermal Rating of Worm Gearing

In the worm gearing, the heat generated due to the work lost in friction must be dissipated in order to avoid over heating of the drive and lubricating oil. The quantity of heat generated (Q_g) is given by

$$Q_g = \text{Power lost in friction in watts} = P(1 - \eta) \quad \text{...}(i)$$

where P = Power transmitted in watts, and

η = Efficiency of the worm gearing.

The heat generated must be dissipated through the lubricating oil to the gear box housing and then to the atmosphere. The heat dissipating capacity depends upon the following factors :

1. Area of the housing (A),

2. Temperature difference between the housing surface and surrounding air ($t_2 - t_1$), and

3. Conductivity of the material (K).

Mathematically, the heat dissipating capacity,

$$Q_d = A(t_2 - t_1)K \quad \text{...}(ii)$$

From equations **(*i*)** and **(*ii*)**, we can find the temperature difference ($t_2 - t_1$). The average value of K may be taken as 378 W/m²/°C.

Notes : 1. The maximum temperature ($t_2 - t_1$) should not exceed 27 to 38°C.

2. The maximum temperature of the lubricant should not exceed 60°C.

3. According to AGMA recommendations, the limiting input power of a plain worm gear unit from the standpoint of heat dissipation, for worm gear speeds upto 2000 r.p.m., may be checked from the following relation, *i.e.*

$$P = \frac{3650\, x^{1.7}}{V.R. + 5}$$

where P = Permissible input power in kW,

x = Centre distance in metres, and

$V.R.$ = Velocity ratio or transmission ratio.

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31.11 Forces Acting on Worm Gears

When the worm gearing is transmitting power, the forces acting on the worm are similar to those on a power screw. Fig. 31.5 shows the forces acting on the worm. It may be noted that the forces on a worm gear are equal in magnitude to that of worm, but opposite in direction to those shown in Fig. 31.5.

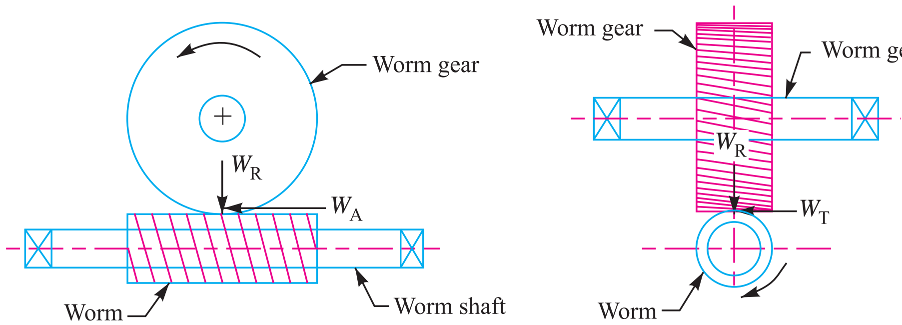

Fig. 31.5. Forces acting on worm teeth.

The various forces acting on the worm may be determined as follows :

1. Tangential force on the worm,

$$W_T = \frac{2 \times \text{Torque on worm}}{\text{Pitch circle diameter of worm } (D_W)}$$

$$= \text{Axial force or thrust on the worm gear}$$

The tangential force (W_T) on the worm produces a twisting moment of magnitude ($W_T \times D_W / 2$) and bends the worm in the horizontal plane.

2. Axial force or thrust on the worm,

$$W_A = W_T / \tan \lambda = \text{Tangential force on the worm gear}$$

$$= \frac{2 \times \text{Torque on the worm gear}}{\text{Pitch circle diameter of worm gear} (D_G)}$$

The axial force on the worm tends to move the worm axially, induces an axial load on the bearings and bends the worm in a vertical plane with a bending moment of magnitude ($W_A \times D_W / 2$).

3. Radial or separating force on the worm,

$$W_R = W_A \,.\, \tan \phi = \text{Radial or separating force on the worm gear}$$

The radial or separating force tends to force the worm and worm gear out of mesh. This force also bends the worm in the vertical plane.

Example 31.2. *A worm drive transmits 15 kW at 2000 r.p.m. to a machine carriage at 75 r.p.m. The worm is triple threaded and has 65 mm pitch diameter. The worm gear has 90 teeth of 6 mm module. The tooth form is to be 20° full depth involute. The coefficient of friction between the mating teeth may be taken as 0.10. Calculate : 1. tangential force acting on the worm ; 2. axial thrust and separating force on worm; and 3. efficiency of the worm drive.*

Solution. Given : P = 15 kW = 15 × 10^3 W ; N_W = 2000 r.p.m. ; N_G = 75 r.p.m. ; n = 3 ; D_W = 65 mm ; T_G = 90 ; m = 6 mm ; ϕ = 20° ; μ = 0.10

1. *Tangential force acting on the worm*

We know that the torque transmitted by the worm

$$= \frac{P \times 60}{2\,\pi\,N_W} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 2000} = 71.6 \text{ N-m} = 71\,600 \text{ N-mm}$$

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∴ Tangential force acting on the worm,

$$W_T = \frac{\text{Torque on worm}}{\text{Radius of worm}} = \frac{71\ 600}{65/2} = 2203 \text{ N} \quad \textbf{Ans.}$$

2. *Axial thrust and separating force on worm*

Let λ = Lead angle.

We know that $\tan \lambda = \dfrac{m \cdot n}{D_W} = \dfrac{6 \times 3}{65} = 0.277$

or $\lambda = \tan^{-1}(0.277) = 15.5°$

∴ Axial thrust on the worm,

$$W_A = W_T / \tan \lambda = 2203 / 0.277 = 7953 \text{ N} \quad \textbf{Ans.}$$

and separating force on the worm

$$W_R = W_A \cdot \tan \phi = 7953 \times \tan 20° = 7953 \times 0.364 = 2895 \text{ N} \quad \textbf{Ans.}$$

3. *Efficiency of the worm drive*

We know that efficiency of the worm drive,

$$\eta = \frac{\tan \lambda (\cos \phi - \mu \cdot \tan \lambda)}{\cos \phi \cdot \tan \lambda + \mu}$$

$$= \frac{\tan 15.5° (\cos 20° - 0.10 \times \tan 15.5°)}{\cos 20° \times \tan 15.5° + 0.10}$$

$$= \frac{0.277 (0.9397 - 0.10 \times 0.277)}{0.9397 \times 0.277 + 0.10} = \frac{0.2526}{0.3603} = 0.701 \text{ or } 70.1\% \quad \textbf{Ans.}$$

31.12 Design of Worm Gearing

In designing a worm and worm gear, the quantities like the power transmitted, speed, velocity ratio and the centre distance between the shafts are usually given and the quantities such as lead angle, lead and number of threads on the worm are to be determined. In order to determine the satisfactory combination of lead angle, lead and centre distance, the following method may be used:

From Fig. 31.6 we find that the centre distance,

$$x = \frac{D_W + D_G}{2}$$

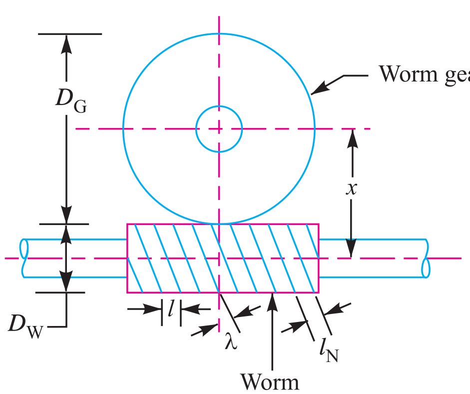

Fig. 31.6. Worm and worm gear.

Worm gear boxes are noted for reliable power transmission.

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The centre distance may be expressed in terms of the axial lead (l), lead angle (λ) and velocity ratio ($V.R.$), as follows :

$$x = \frac{l}{2\pi}(\cot \lambda + V.R.)$$

In terms of normal lead ($l_N = l \cos \lambda$), the above expression may be written as :

$$x = \frac{l_N}{2\pi}\left(\frac{1}{\sin \lambda} + \frac{V.R.}{\cos \lambda}\right)$$

or

$$\frac{x}{l_N} = \frac{1}{2\pi}\left(\frac{1}{\sin \lambda} + \frac{V.R.}{\cos \lambda}\right) \quad ...(i)$$

Since the velocity ratio ($V.R.$) is usually given, therefore the equation **(*i*)** contains three variables *i.e.* x, l_N and λ. The right hand side of the above expression may be calculated for various values of velocity ratios and the curves are plotted as shown in Fig. 31.7. The lowest point on each of the curves gives the lead angle which corresponds to the minimum value of x / l_N. This minimum value represents the minimum centre distance that can be used with a given lead or inversely the maximum lead that can be used with a given centre distance. Now by using Table 31.2 and standard modules, we can determine the combination of lead angle, lead, centre distance and diameters for the given design specifications.

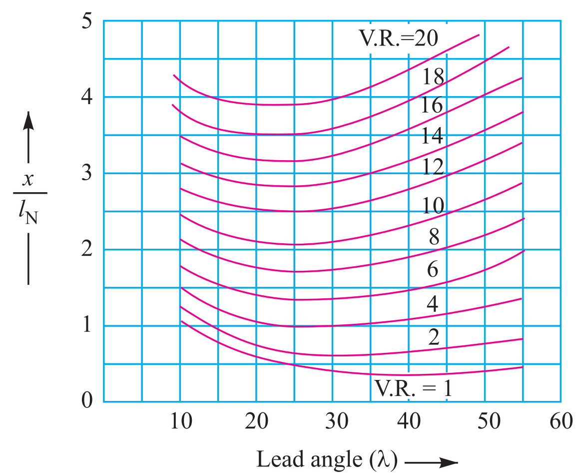


Fig. 31.7. Worm gear design curves.

Note : The lowest point on the curve may be determined mathematically by differentiating the equation **(*i*)** with respect to λ and equating to zero, *i.e.*

$$\frac{(V.R.)\sin^3 \lambda - \cos^3 \lambda}{\sin^2 \lambda . \cos^2 \lambda} = 0 \quad \text{or} \quad V.R. = \cot^3 \lambda$$

Example 31.3. *Design 20° involute worm and gear to transmit 10 kW with worm rotating at 1400 r.p.m. and to obtain a speed reduction of 12 : 1. The distance between the shafts is 225 mm.*

Solution. Given : $\phi = 20°$; $P = 10$ kW = 10 000 W ; $N_W = 1400$ r.p.m. ; $V.R. = 12$; $x = 225$ mm

The worm and gear is designed as discussed below :

1. *Design of worm*

Let l_N = Normal lead, and

λ = Lead angle.

Top

Worm gear of a steering mechanism in an automobile.

We have discussed in Art. 31.12 that the value of x / l_N will be minimum corresponding to

$$\cot^3 \lambda = V.R. = 12 \quad \text{or} \quad \cot \lambda = 2.29$$

$$\therefore \quad \lambda = 23.6°$$

We know that
$$\frac{x}{l_N} = \frac{1}{2\pi}\left(\frac{1}{\sin\lambda} + \frac{V.R.}{\cos\lambda}\right)$$

$$\frac{225}{l_N} = \frac{1}{2\pi}\left(\frac{1}{\sin 23.6°} + \frac{12}{\cos 23.6°}\right) = \frac{1}{2\pi}(2.5 + 13.1) = 2.5$$

$$\therefore \quad l_N = 225 / 2.5 = 90 \text{ mm}$$

and axial lead, $l = l_N / \cos\lambda = 90 / \cos 23.6° = 98.2$ mm

From Table 31.2, we find that for a velocity ratio of 12, the number of starts or threads on the worm,

$$n = T_W = 4$$

∴ Axial pitch of the threads on the worm,

$$p_a = l / 4 = 98.2 / 4 = 24.55 \text{ mm}$$

$$\therefore \quad m = p_a / \pi = 24.55 / \pi = 7.8 \text{ mm}$$

Let us take the standard value of module, $m = 8$ mm

∴ Axial pitch of the threads on the worm,

$$p_a = \pi m = p \times 8 = 25.136 \text{ mm}$$ **Ans.**

Axial lead of the threads on the worm,

$$l = p_a \cdot n = 25.136 \times 4 = 100.544 \text{ mm}$$ **Ans.**

and normal lead of the threads on the worm,

$$l_N = l \cos\lambda = 100.544 \cos 23.6° = 92 \text{ mm}$$ **Ans.**

We know that the centre distance,

$$x = \frac{l_N}{2\pi}\left(\frac{1}{\sin\lambda} + \frac{V.R.}{\cos\lambda}\right) = \frac{92}{2\pi}\left(\frac{1}{\sin 23.6°} + \frac{12}{\cos 23.6°}\right)$$

$$= 14.64 (2.5 + 13.1) = 230 \text{ mm}$$ **Ans.**

Let D_W = Pitch circle diameter of the worm.

We know that
$$\tan\lambda = \frac{l}{\pi D_W}$$

$$\therefore \quad D_W = \frac{l}{\pi \tan\lambda} = \frac{100.544}{\pi \tan 23.6°} = 73.24 \text{ mm}$$ **Ans.**

Since the velocity ratio is 12 and the worm has quadruple threads (*i.e.* $n = T_W = 4$), therefore number of teeth on the worm gear,

$$T_G = 12 \times 4 = 48$$

From Table 31.3, we find that the face length of the worm or the length of threaded portion is

$$L_W = p_c\,(4.5 + 0.02\,T_W)$$
$$= 25.136\,(4.5 + 0.02 \times 4) = 115 \text{ mm} \qquad ...(\because\ p_c = p_a)$$

This length should be increased by 25 to 30 mm for the feed marks produced by the vibrating grinding wheel as it leaves the thread root. Therefore let us take

$$L_W = 140 \text{ mm}$$ **Ans.**

We know that depth of tooth,

$$h = 0.623\,p_c = 0.623 \times 25.136 = 15.66 \text{ mm}$$ **Ans.**

...(From Table 31.3)

and addendum, $a = 0.286\,p_c = 0.286 \times 25.136 = 7.2$ mm **Ans.**

∴ Outside diameter of worm,

$$D_{OW} = D_W + 2a = 73.24 + 2 \times 7.2 = 87.64 \text{ mm}$$ **Ans.**

2. *Design of worm gear*

We know that pitch circle diameter of the worm gear,

$$D_G = m\,.\,T_G = 8 \times 48 = 384 \text{ mm} = 0.384 \text{ m}$$ **Ans.**

From Table 31.4, we find that outside diameter of worm gear,

$$D_{OG} = D_G + 0.8903\,p_c = 384 + 0.8903 \times 25.136 = 406.4 \text{ mm}$$ **Ans.**

Throat diameter,

$$D_T = D_G + 0.572\,p_c = 384 + 0.572 \times 25.136 = 398.4 \text{ mm}$$ **Ans.**

and face width, $b = 2.15\,p_c + 5 \text{ mm} = 2.15 \times 25.136 + 5 = 59$ mm **Ans.**

Let us now check the designed worm gearing from the standpoint of tangential load, dynamic load, static load or endurance strength, wear load and heat dissipation.

(a) Check for the tangential load

Let N_G = Speed of the worm gear in r.p.m.

We know that velocity ratio of the drive,

$$V.R. = \frac{N_W}{N_G} \quad \text{or} \quad N_G = \frac{N_W}{V.R.} = \frac{1400}{12} = 116.7 \text{ r.p.m}$$

∴ Torque transmitted,

$$T = \frac{P \times 60}{2\,\pi\,N_G} = \frac{10\,000 \times 60}{2\,\pi \times 116.7} = 818.2 \text{ N-m}$$

and tangential load acting on the gear,

$$W_T = \frac{2 \times \text{Torque}}{D_G} = \frac{2 \times 818.2}{0.384} = 4260 \text{ N}$$

We know that pitch line or peripheral velocity of the worm gear,

$$v = \frac{\pi\,.\,D_G\,.\,N_G}{60} = \frac{\pi \times 0.384 \times 116.7}{60} = 2.35 \text{ m/s}$$

∴ Velocity factor,

$$C_v = \frac{6}{6 + v} = \frac{6}{6 + 2.35} = 0.72$$

Gears are usually enclosed in boxes to protect them from environmental pollution and provide them proper lubrication.

and tooth form factor for 20° involute teeth,

$$y = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{48} = 0.135$$

Since the worm gear is generally made of phosphor bronze, therefore taking the allowable static stress for phosphor bronze, σ_o = 84 MPa or N/mm^2.

We know that the designed tangential load,

$$W_T = (\sigma_o \,.\, C_v)\, b.\, \pi\, m\,.\, y = (84 \times 0.72)\ 59 \times \pi \times 8 \times 0.135 \text{ N}$$
$$= 12\ 110 \text{ N}$$

Since this is more than the tangential load acting on the gear (*i.e.* 4260 N), therefore the design is safe from the standpoint of tangential load.

(b) Check for dynamic load

We know that the dynamic load,

$$W_D = W_T / C_v = 12\ 110 / 0.72 = 16\ 820 \text{ N}$$

Since this is more than W_T = 4260 N, therefore the design is safe from the standpoint of dynamic load.

(c) Check for static load or endurance strength

We know that the flexural endurance limit for phosphor bronze is

$$\sigma_e = 168 \text{ MPa or N/mm}^2$$

∴ Static load or endurance strength,

$$W_S = \sigma_e\,.\, b.\, \pi\, m\,.\, y = 168 \times 59 \times \pi \times 8 \times 0.135 = 33\ 635 \text{ N}$$

Since this is much more than W_T = 4260 N, therefore the design is safe from the standpoint of static load or endurance strength.

(d) Check for wear

Assuming the material for worm as hardened steel, therefore from Table 31.5, we find that for hardened steel worm and phosphor bronze worm gear, the value of load stress factor,

$$K = 0.55 \text{ N/mm}^2$$

∴ Limiting or maximum load for wear,

$$W_W = D_G \,.\, b \,.\, K = 384 \times 59 \times 0.55 = 12\ 461 \text{ N}$$

Since this is more than W_T = 4260 N, therefore the design is safe from the standpoint of wear.

(e) Check for heat dissipation

First of all, let us find the efficiency of the worm gearing (η).

We know that rubbing velocity,

$$v_r = \frac{\pi\, D_W \,.\, N_W}{\cos \lambda} = \frac{\pi \times 0.07324 \times 1400}{\cos 23.6°} = 351.6 \text{ m / min}$$

...(D_W is taken in metres)

∴ Coefficient of friction,

$$\mu = 0.025 + \frac{v_r}{18\ 000} = 0.025 + \frac{351.6}{18\ 000} = 0.0445$$

...(∴ v_r is greater than 180 m/min)

and angle of friction, $\phi_1 = \tan^{-1} \mu = \tan^{-1} (0.0445) = 2.548°$

We know that efficiency,

$$\eta = \frac{\tan \lambda}{\tan (\lambda + \phi_1)} = \frac{\tan 23.6°}{\tan (23.6 + 2.548)} = \frac{0.4369}{0.4909} = 0.89 \text{ or } 89\%$$

Assuming 25 per cent overload, heat generated,

$$Q_g = 1.25\, P\, (1 - \eta) = 1.25 \times 10\ 000\ (1 - 0.89) = 1375 \text{ W}$$

We know that projected area of the worm,

$$A_W = \frac{\pi}{4} (D_W)^2 = \frac{\pi}{4} (73.24)^2 = 4214 \text{ mm}^2$$

and projected area of the worm gear,

$$A_G = \frac{\pi}{4} (D_G)^2 = \frac{\pi}{4} (384)^2 = 115\ 827 \text{ mm}^2$$

∴ Total projected area of worm and worm gear,

$$A = A_W + A_G = 4214 + 115\ 827 = 120\ 041 \text{ mm}^2$$

$$= 120\ 041 \times 10^{-6} \text{ m}^2$$

We know that heat dissipating capacity,

$$Q_d = A\,(t_2 - t_1)\, K = 120\ 041 \times 10^{-6}\,(t_2 - t_1)\, 378 = 45.4\,(t_2 - t_1)$$

The heat generated must be dissipated in order to avoid over heating of the drive, therefore equating $Q_g = Q_d$, we have

$$t_2 - t_1 = 1375 / 45.4 = 30.3°\text{C}$$

Since this temperature difference $(t_2 - t_1)$ is within safe limits of 27 to 38°C, therefore the design is safe from the standpoint of heat.

3. Design of worm shaft

Let d_W = Diameter of worm shaft.

We know that torque acting on the worm gear shaft,

$$T_{gear} = \frac{1.25\, P \times 60}{2\, \pi\, N_G} = \frac{1.25 \times 10\,000 \times 60}{2\, \pi \times 116.7} = 1023 \text{ N-m}$$

$$= 1023 \times 10^3 \text{ N-mm}$$

...(Taking 25% overload)

∴ Torque acting on the worm shaft,

$$T_{worm} = \frac{T_{gear}}{V.R. \times \eta} = \frac{1023}{12 \times 0.89} = 96 \text{ N-m} = 96 \times 10^3 \text{ N-mm}$$

Differential inside an automobile.

We know that tangential force on the worm,

$$W_T = \text{Axial force on the worm gear}$$

$$= \frac{2 \times T_{worm}}{D_W} = \frac{2 \times 96 \times 10^3}{73.24} = 2622 \text{ N}$$

Axial force on the worm,

$$W_A = \text{Tangential force on the worm gear}$$

$$= \frac{2 \times T_{gear}}{D_G} = \frac{2 \times 1023 \times 10^3}{384} = 5328 \text{ N}$$

and radial or separating force on the worm

$$W_R = \text{Radial or separating force on the worm gear}$$

$$= W_A \cdot \tan \phi = 5328 \times \tan 20° = 1940 \text{ N}$$

Let us take the distance between the bearings of the worm shaft (x_1) equal to the diameter of the worm gear (D_G), *i.e.*

$$x_1 = D_G = 384 \text{ mm}$$

∴ Bending moment due to the radial force (W_R) in the vertical plane

$$= \frac{W_R \times x_1}{4} = \frac{1940 \times 384}{4} = 186\,240 \text{ N-mm}$$

and bending moment due to axial force (W_A) in the vertical plane

$$= \frac{W_A \times D_W}{4} = \frac{5328 \times 73.24}{4} = 97\,556 \text{ N-mm}$$

∴ Total bending moment in the vertical plane,

$$M_1 = 186\,240 + 97\,556 = 283\,796 \text{ N-mm}$$

We know that bending moment due to tangential force (W_T) in the horizontal plane,

$$M_2 = \frac{W_T \times D_G}{4} = \frac{2622 \times 384}{4} = 251\,712 \text{ N-mm}$$

∴ Resultant bending moment on the worm shaft,

$$M_{worm} = \sqrt{(M_1)^2 + (M_2)^2} = \sqrt{(283\,796)^2 + (251\,712)^2} = 379\,340 \text{ N-mm}$$

We know that equivalent twisting moment on the worm shaft,

$$T_{ew} = \sqrt{(T_{worm})^2 + (M_{worm})^2} = \sqrt{(96 \times 10^3)^2 + (379\ 340)^2} \text{ N-mm}$$

$$= 391\ 300 \text{ N-mm}$$

We also know that equivalent twisting moment (T_{ew}),

$$391\ 300 = \frac{\pi}{16} \times \tau\ (d_W)^3 = \frac{\pi}{16} \times 50\ (d_W)^3 = 9.82\ (d_W)^3$$

...(Taking τ = 50 MPa or N/mm^2)

∴ $(d_W)^3 = 391\ 300 / 9.82 = 39\ 850$ or $d_W = 34.2$ say 35 mm **Ans.**

Let us now check the maximum shear stress induced.

We know that the actual shear stress,

$$\tau = \frac{16\ T_{ew}}{\pi\ (d_W)^3} = \frac{16 \times 391\ 300}{\pi\ (35)^3} = 46.5 \text{ N/mm}^2$$

and direct compressive stress on the shaft due to the axial force,

$$\sigma_c = \frac{W_A}{\frac{\pi}{4}\ (d_W)^2} = \frac{5328}{\frac{\pi}{4}\ (35)^2} = 5.54 \text{ N/mm}^2$$

∴ Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\ \tau^2} = \frac{1}{2}\sqrt{(5.54)^2 + 4\ (46.5)^2} = 46.6 \text{ MPa}$$

Since the maximum shear stress induced is less than 50 MPa (assumed), therefore the design of worm shaft is satisfactory.

4. Design of worm gear shaft

Let d_G = Diameter of worm gear shaft.

We have calculated above that the axial force on the worm gear

= 2622 N

Tangential force on the worm gear

= 5328 N

and radial or separating force on the worm gear

= 1940 N

We know that bending moment due to the axial force on the worm gear

$$= \frac{\text{Axial force} \times D_G}{4} = \frac{2622 \times 384}{4} = 251\ 712 \text{ N-mm}$$

The bending moment due to the axial force will be in the vertical plane.

Let us take the distance between the bearings of the worm gear shaft (x_2) as 250 mm.

∴ Bending moment due to the radial force on the worm gear

$$= \frac{\text{Radial force} \times x_2}{4} = \frac{1940 \times 250}{4} = 121\ 250 \text{ N-mm}$$

The bending moment due to the radial force will also be in the vertical plane.

∴ Total bending moment in the vertical plane

$$M_3 = 251\ 712 + 121\ 250 = 372\ 962 \text{ N-mm}$$

We know that the bending moment due to the tangential force in the horizontal plane

$$M_4 = \frac{\text{Tangential force} \times x_2}{4} = \frac{5328 \times 250}{4} = 333\ 000 \text{ N-mm}$$

∴ Resultant bending moment on the worm gear shaft,

$$M_{gear} = \sqrt{(M_3)^2 + (M_4)^2} = \sqrt{(372\ 962)^2 + (333\ 000)^2} \text{ N-mm}$$

$$= 500 \times 10^3 \text{ N-mm}$$

We have already calculated that the torque acting on the worm gear shaft,

$$T_{gear} = 1023 \times 10^3 \text{ N-mm}$$

∴ Equivalent twisting moment on the worm gear shaft,

$$T_{eg} = \sqrt{(T_{gear})^2 + (M_{gear})^2} = \sqrt{(1023 \times 10^3)^2 + (500 \times 10^3)^2} \text{ N-mm}$$

$$= 1.14 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_{eg}),

$$1.14 \times 10^6 = \frac{\pi}{16} \times \tau\,(d_G)^3 = \frac{\pi}{16} \times 50\,(d_G)^3 = 9.82\,(d_G)^3$$

$$\therefore \quad (d_G)^3 = 1.14 \times 10^6 / 9.82 = 109 \times 10^3$$

or $d_G = 48.8$ say 50 mm **Ans.**

Let us now check the maximum shear stress induced.

We know that actual shear stress,

$$\tau = \frac{16\,T_{eg}}{\pi\,(d_G)^3} = \frac{16 \times 1.14 \times 10^6}{\pi\,(50)^3} = 46.4 \text{ N/mm}^2 = 46.4 \text{ MPa}$$

and direct compressive stress on the shaft due to the axial force,

$$\sigma_c = \frac{\text{Axial force}}{\frac{\pi}{4}(d_G)^2} = \frac{2622}{\frac{\pi}{4}(50)^2} = 1.33 \text{ N/mm}^3 = 1.33 \text{ MPa}$$

∴ Maximum shear stress,

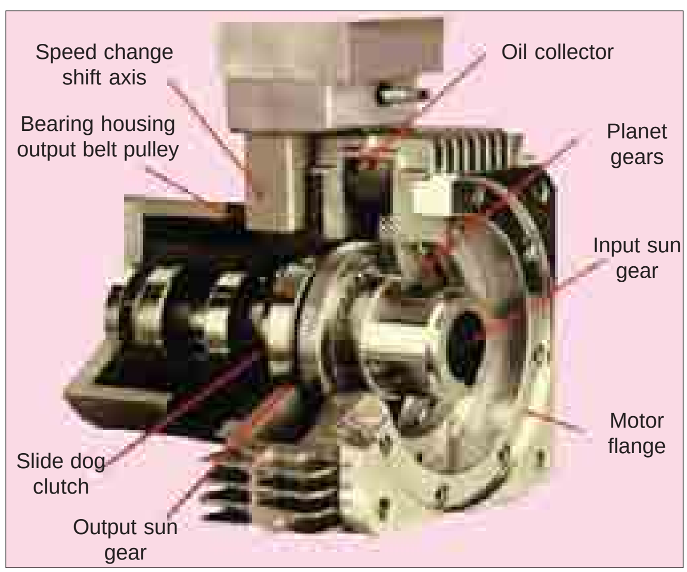
$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(1.33)^2 + 4\,(46.4)^2} = 46.4 \text{ MPa}$$

Since the maximum shear stress induced is less than 50 MPa (assumed), therefore the design for worm gear shaft is satisfactory.

Example 31.4. *A speed reducer unit is to be designed for an input of 1.1 kW with a transmission ratio 27. The speed of the hardened steel worm is 1440 r.p.m. The worm wheel is to be made of phosphor bronze. The tooth form is to be 20° involute.*

Solution. Given : $P = 1.1$ kW $= 1100$ W ; $V.R. = 27$; $N_W = 1440$ r.p.m. ; $\phi = 20°$

A speed reducer unit (*i.e.*, worm and worm gear) may be designed as discussed below.



Sun and Planet gears.

Since the centre distance between the shafts is not known, therefore let us assume that for this size unit, the centre distance (x) = 100 mm.

We know that pitch circle diameter of the worm,

$$D_W = \frac{(x)^{0.875}}{1.416} = \frac{(100)^{0.875}}{1.416} = 39.7 \text{ say } 40 \text{ mm}$$

∴ Pitch circle diameter of the worm gear,

$$D_G = 2x - D_W = 2 \times 100 - 40 = 160 \text{ mm}$$

From Table 31.2, we find that for the transmission ratio of 27, we shall use double start worms.

∴ Number of teeth on the worm gear,

$$T_G = 2 \times 27 = 54$$

We know that the axial pitch of the threads on the worm (p_a) is equal to circular pitch of teeth on the worm gear (p_c).

$$\therefore \quad p_a = p_c = \frac{\pi\, D_G}{T_G} = \frac{\pi \times 160}{54} = 9.3 \text{ mm}$$

and module,

$$m = \frac{p_c}{\pi} = \frac{9.3}{\pi} = 2.963 \text{ say } 3 \text{ mm}$$

∴ Actual circular pitch,

$$p_c = \pi\, m = \pi \times 3 = 9.426 \text{ mm}$$

Actual pitch circle diameter of the worm gear,

$$D_G = \frac{p_c \,.T_G}{\pi} = \frac{9.426 \times 54}{\pi} = 162 \text{ mm} \quad \textbf{Ans.}$$

and actual pitch circle diameter of the worm,

$$D_W = 2x - D_G = 2 \times 100 - 162 = 38 \text{ mm} \quad \textbf{Ans.}$$

The face width of the worm gear (b) may be taken as 0.73 times the pitch circle diameter of worm (D_W).

$$\therefore \quad b = 0.73\, D_W = 0.73 \times 38 = 27.7 \text{ say } 28 \text{ mm}$$

Let us now check the design from the standpoint of tangential load, dynamic load, static load or endurance strength, wear load and heat dissipation.

1. *Check for the tangential load*

Let N_G = Speed of the worm gear in r.p.m.

We know that velocity ratio of the drive,

$$V.R. = \frac{N_W}{N_G} \text{ or } N_G = \frac{N_W}{V.R.} = \frac{1440}{27} = 53.3 \text{ r.p.m}$$

∴ Peripheral velocity of the worm gear,

$$v = \frac{\pi\, D_G \,.\, N_G}{60} = \frac{\pi \times 0.162 \times 53.3}{60} = 0.452 \text{ m/s}$$

... (D_G is taken in metres)

and velocity factor,

$$C_v = \frac{6}{6 + v} = \frac{6}{6 + 0.452} = 0.93$$

We know that for 20° involute teeth, the tooth form factor,

$$y = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{54} = 0.137$$

From Table 31.4, we find that allowable static stress for phosphor bronze is

$$\sigma_o = 84 \text{ MPa or N/mm}^2$$

∴ Tangential load transmitted,

$$W_T = (\sigma_o \,.\, C_v)\, b \,.\, \pi\, m \,.\, y = (84 \times 0.93)\, 28 \times \pi \times 3 \times 0.137 \text{ N}$$
$$= 2825 \text{ N}$$

and power transmitted due to the tangential load,

$$P = W_T \times v = 2825 \times 0.452 = 1277 \text{ W} = 1.277 \text{ kW}$$

Since this power is more than the given power to be transmitted (1.1 kW), therefore the design is safe from the standpoint of tangential load.

2. Check for the dynamic load

We know that the dynamic load,

$$W_D = W_T / C_v = 2825 / 0.93 = 3038 \text{ N}$$

and power transmitted due to the dynamic laod,

$$P = W_D \times v = 3038 \times 0.452 = 1373 \text{ W} = 1.373 \text{ kW}$$

Since this power is more than the given power to be transmitted, therefore the design is safe from the standpoint of dynamic load.

3. Check for the static load or endurance strength

From Table 31.8, we find that the flexural endurance limit for phosphor bronze is

$$\sigma_e = 168 \text{ MPa or N/mm}^2$$

∴ Static load or endurance strength,

$$W_S = \sigma_e \,.\, b.\, \pi\, m \,.\, y = 168 \times 28 \times \pi \times 3 \times 0.137 = 6075 \text{ N}$$

and power transmitted due to the static load,

$$P = W_S \times v = 6075 \times 0.452 = 2746 \text{ W} = 2.746 \text{ kW}$$

Since this power is more than the power to be transmitted (1.1 kW), therefore the design is safe from the standpoint of static load.

4. Check for the wear load

From Table 31.5, we find that the load stress factor for hardened steel worm and phosphor bronze worm gear is

$$K = 0.55 \text{ N/mm}^2$$

∴ Limiting or maximum load for wear,

$$W_W = D_G \,.\, b \,.\, K = 162 \times 28 \times 0.55 = 2495 \text{ N}$$

and power transmitted due to the wear load,

$$P = W_W \times v = 2495 \times 0.452 = 1128 \text{ W} = 1.128 \text{ kW}$$

Since this power is more than the given power to be transmitted (1.1 kW), therefore the design is safe from the standpoint of wear.

5. Check for the heat dissipation

We know that permissible input power,

$$P = \frac{3650\,(x)^{1.7}}{V.R + 5} = \frac{3650\,(0.1)^{1.7}}{27 + 5} = 2.27 \text{ kW} \qquad \text{... (}x\text{ is taken in metres)}$$

Since this power is more than the given power to be transmitted (1.1 kW), therefore the design is safe from the standpoint of heat dissipation.

EXERCISES

1. A double threaded worm drive is required for power transmission between two shafts having their axes at right angles to each other. The worm has 14½° involute teeth. The centre distance is approximately 200 mm. If the axial pitch of the worm is 30 mm and lead angle is 23°, find 1. lead; 2. pitch circle diameters of worm and worm gear; 3. helix angle of the worm; and 4. efficiency of the drive if the coefficient of friction is 0.05. **[Ans. 60 mm ; 45 mm ; 355 mm ; 67° ; 87.4%]**

The worm in its place. One can also see the two cubic worm bearing blocks and the big gear.

2. A double threaded worm drive has an axial pitch of 25 mm and a pitch circle diameter of 70 mm. The torque on the worm gear shaft is 1400 N-m. The pitch circle diameter of the worm gear is 250 mm and the tooth pressure angle is 25°. Find : 1. tangential force on the worm gear, 2. torque on the worm shaft, 3. separating force on the worm, 4. velocity ratio, and 5. efficiency of the drive, if the coefficient of friction between the worm thread and gear teeth is 0.04.

   [Ans. 11.2 kN ; 88.97 N-m ; 5220 N ; 82.9%]

3. Design a speed reducer unit of worm and worm wheel for an input of 1 kW with a transmission ratio of 25. The speed of the worm is 1600 r.p.m. The worm is made of hardened steel and wheel of phosphor bronze for which the material combination factor is 0.7 N/mm^2. The static stress for the wheel material is 56 MPa. The worm is made of double start and the centre distance between the axes of the worm and wheel is 120 mm. The tooth form is to be 14½° involute. Check the design for strength, wear and heat dissipation.

4. Design worm and gear speed reducer to transmit 22 kW at a speed of 1440 r.p.m. The desired velocity ratio is 24 : 1. An efficiency of atleast 85% is desired. Assume that the worm is made of hardened steel and the gear of phosphor bronze.

QUESTIONS

1. Discuss, with neat sketches, the various types of worms and worm gears.
2. Define the following terms used in worm gearing :

   (*a*) Lead; (*b*) Lead angle; (*c*) Normal pitch; and (*d*) Helix angle.
3. What are the various forces acting on worm and worm gears ?
4. Write the expression for centre distance in terms of axial lead, lead angle and velocity ratio.

OBJECTIVE TYPE QUESTIONS

1. The worm gears are widely used for transmitting power at .... velocity ratios between non-intersecting shafts.

(*a*) high (*b*) low

2. In worm gears, the angle between the tangent to the thread helix on the pitch cylinder and the plane normal to the axis of worm is called

(*a*) pressure angle (*b*) lead angle

(*c*) helix angle (*d*) friction angle

3. The normal lead, in a worm having multiple start threads, is given by

(*a*) $l_N = l / \cos \lambda$ (*b*) $l_N = l \, . \cos \lambda$

(*c*) $l_N = l$ (*d*) $l_N = l \tan$

where l_N = Normal lead,

l = Lead, and

λ = Lead angle.

4. The number of starts on the worm for a velocity ratio of 40 should be

(*a*) single (*b*) double

(*c*) triple (*d*) quadruple

5. The axial thrust on the worm (W_A) is given by

(*a*) $W_A = W_T \, . \tan \phi$ (*b*) $W_A = W_T / \tan \phi$

(*c*) $W_A = W_T \, . \tan \lambda$ (*d*) $W_A = W_T / \tan \lambda$

where W_T = Tangential force acting on the worm,

ϕ = Pressure angle, and

λ = Lead angle.

ANSWERS

1. (*a*) **2.** (*b*) **3.** (*b*) **4.** (*a*) **5.** (*d*)